

Introduction of Theano (1)

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Introduction

- Theano is a Python library that lets you to define, optimize, and evaluate mathematical expressions.
 - Especially useful for machine learning
- Prerequisite: python and numpy
 - <http://cs231n.github.io/python-numpy-tutorial/>
- Target of this class:
 - Introduce theano from the beginning, so you can build your own DNN by it

Installation

Install Theano

- How to install Theano
 - <http://deeplearning.net/software/theano/install.html#install>
- Make sure there is no error message when you “import theano” in python
 - E.g.

```
xnor@Speech-GTX-780 ★ python
Python 2.7.6 (default, Mar 22 2014, 22:59:56)
[GCC 4.8.2] on linux2
Type "help", "copyright", "credits" or "license" for more information.
>>> import theano
>>> █
```

Basic

Review Machine Learning

- **Define a function set (Model):** $f(x; w)$
 - x : input
 - w : model parameters
- **Define what is the best function:** Define a cost function $C(f)$
- **Pick the best function by data:** Training
 - In deep learning, this is usually done by gradient descent.

Power of Theano

- After defining a cost function, Theano can automatically compute the gradients.
- To use Theano in deep learning you only have to learn
 - How to define a function
 - How to compute the gradient
- Then that's it.

Define function - Overview

- E.g. Define a function $f(x) = x^2$, then compute $f(-2)$

```
1 import theano
2
3 x = theano.tensor.scalar()
4 y = x ** 2
5 f = theano.function([x], y)
6
7 print f(-2)
8
```

Type the left code in a file name “xxx.py”

Execute “python xxx.py”

You should get 4

Step 0. Declare that you want to use Theano (line 1)

Step 1. Define input variable x (line 3)

Step 2. Define output variable y (line 4)

Step 3. Declare the function as f (line 5)

Step 4. Use the function f (line 7)

Define function - Overview

- E.g. Define a function $f(x) = x^2$, then compute $f(-2)$

```
1 import theano  
2  
3 x = theano.tensor.scalar()  
4 y = x ** 2  
5 f = theano.function([x], y)  
6  
7 print f(-2)  
8
```

||

```
1 def f(x):  
2     return x ** 2  
3  
4 print f(-2)  
5  
6
```

Type the left code in a file name “xxx.py”

Execute “python xxx.py”

You should get 4

So why we define a function by Theano.

It will be clear when we compute the gradients.

Define function

– Step1. Define Input Variables

- A variable can be a scalar, a matrix or a tensor

```
1 import theano
2
3 a = theano.tensor.scalar()
4 b = theano.tensor.matrix()
5 c = theano.tensor.matrix('ha ha ha')
6
7 print a
8 print b
9 print c
```

Line 3: declare a scalar a

Line 4: declare a matrix b

Line 5: declare a matrix c with name “ha ha ha”

The name of a variable only make difference when you try to print the variable.

Define function

– Step1. Define Input Variables

- A variable can be a scalar or a matrix

```
1 import theano
2
3 a = theano.tensor.scalar()
4 b = theano.tensor.matrix()
5 c = theano.tensor.matrix('ha ha ha')
6
7 print a } 
8 print b }
9 print c }
```

Line 7,8,9: let's print the three variables a, b, c to see what we get

```
<TensorType(float64, scalar)>
<TensorType(float64, matrix)>
ha ha ha
```

a, b, c are symbols without any values

Define function

- Step1. Define Input Variables

- A variable can be a scalar or a matrix

```
1 import theano
2
3 a = theano.tensor.scalar()
4 b = theano.tensor.matrix()
5 c = theano.tensor.matrix('ha ha ha')
6
7 print a
8 print b
9 print c
```

||

```
1 import theano
2 import theano.tensor as T
3
4 a = T.scalar()
5 b = T.matrix()
6 c = T.matrix('ha ha ha')
7
```

simplification

Define function

– Step2. Define Output Variables

- Output variables are defined based on their relations with the input variables
- Below are some examples

```
3 x1 = T.scalar()  
4 x2 = T.scalar()  
5 x3 = T.matrix()  
6 x4 = T.matrix()  
7  
8 y1 = x1 + x2  
9  
10 y2 = x1 * x2  
11  
12 y3 = x3 * x4  
13  
14 y4 = T.dot( x3 , x4 )  
15
```

y1 equals to x1 plus x2

y2 equals to x1 times x2

y3 is the *elementwise* multiplication of x3 and x4

y4 is the matrix multiplication of x3 and x4

Define function

– Step 3. Declare Function

```
f = theano.function([x], y)
```

Declare the
function as f

Function
input

Function
output

Note: the input of a function should be a list.
That is, always put the input in “[]”

```
f = theano.function(  
    inputs=[x],  
    outputs=y  
)
```

Define the function input and
output explicitly.
(equivalent to the above
usage)

Define function

– Step 3. Declare Function

```
1 import theano
2 import theano.tensor as T
3
4 x1 = T.scalar()
5 x2 = T.scalar()
6
7 y1 = x1 * x2
8 y2 = x1 ** 2 + x2 ** 0.5
9
10 f = theano.function([x1, x2], [y1, y2])
11
12 z = f(2, 4)
13 print z
```

Define function

– Step 4. Use Function

```
1 import theano
2 import theano.tensor as T
3
4 x1 = T.scalar()
5 x2 = T.scalar()
6
7 y1 = x1 * x2
8 y2 = x1 ** 2 + x2 ** 0.5
9
10 f = theano.function([x1, x2], [y1, y2])
11
12 z = f(2, 4)
13 print z
```

Line 12: simply use the function `f` you declared as a normal python function

Line 13: Print the function output -> `[array(8.0), array(6.0)]`
(The theano function output is a numpy.ndarray.)

Define function

- Examples for Matrix

```
1 import theano
2 import theano.tensor as T
3
4 a = T.matrix()
5 b = T.matrix()
6
7 c = a * b
8 d = T.dot( a , b )
9
10 F1 = theano.function([a, b],c)
11 F2 = theano.function([a, b],d)
12
13 A = [[1, 2],[3, 4]]
14 B = [[2, 4],[6, 8]]
15 C = [[1, 2],[3, 4],[5, 6]]
16
17 print F1( A , B )
18 print F2( C , B )
```

Output:

```
[ [ 2.    8. ]
  [ 18.   32. ] ]
[ [ 14.    20. ]
  [ 30.   44. ] ]
[ 46.   68. ] ]
```

Be careful that the dimensions of the input matrices should be correct.

Compute Gradients

- Computing the gradients with respect to a variable is so simple.
- Given a function with input variable x and output variable y
 - To compute dy/dx , simply $g = T.grad(y , x)$
 - Note: To compute the gradient, y should be a scalar.
- That's it!

Compute Gradients

- Example 1

```
1 import theano
2 import theano.tensor as T
3
4 x = T.scalar('x')
5 y = x ** 2
6 g = T.grad( y , x ) g = dy/dx = 2x
7
8 f = theano.function( [x] , y )
9 f_prime = theano.function( [x] , g )
10
11 print f(-2)
12 print f_prime(-2) output “-4”
```

Compute Gradients

- Example 2

```
1 import theano
2 import theano.tensor as T
3
4 x1 = T.scalar()
5 x2 = T.scalar()
6
7 y = x1 * x2
8 g = T.grad(y, [x1, x2]) →
9
10 f = theano.function([x1, x2], y)
11 f_prime = theano.function([x1, x2], g)
12
13 print f(2, 4)
14 print f_prime(2, 4)
```

compute the gradients with
respect to multiple variables

$\begin{aligned} g &= [\partial y / \partial x_1, \partial y / \partial x_2] \\ &= [x_2, x_1] \end{aligned}$

Compute Gradients

- Example 3

$$\text{If } A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}$$

$$\text{If } B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$$

(Note that the dimensions of A and B is not necessary 2 X 2.
Here is just an example.)

$$C = \begin{bmatrix} a_1b_1 & a_2b_2 \\ a_3b_3 & a_4b_4 \end{bmatrix} \text{ (line 7)}$$

$$D = a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4 \text{ (line 8)}$$

```
1 import theano
2 import theano.tensor as T
3
4 A = T.matrix()
5 B = T.matrix()
6
7 C = A * B
8 D = T.sum(C)
9
10 g = T.grad( D , A )
11
12 y_prime = theano.function( [A, B] , g )
13
14 A = [[1, 2], [3, 4]]
15 B = [[2, 4], [6, 8]]
16
17 print y_prime(A, B)
```

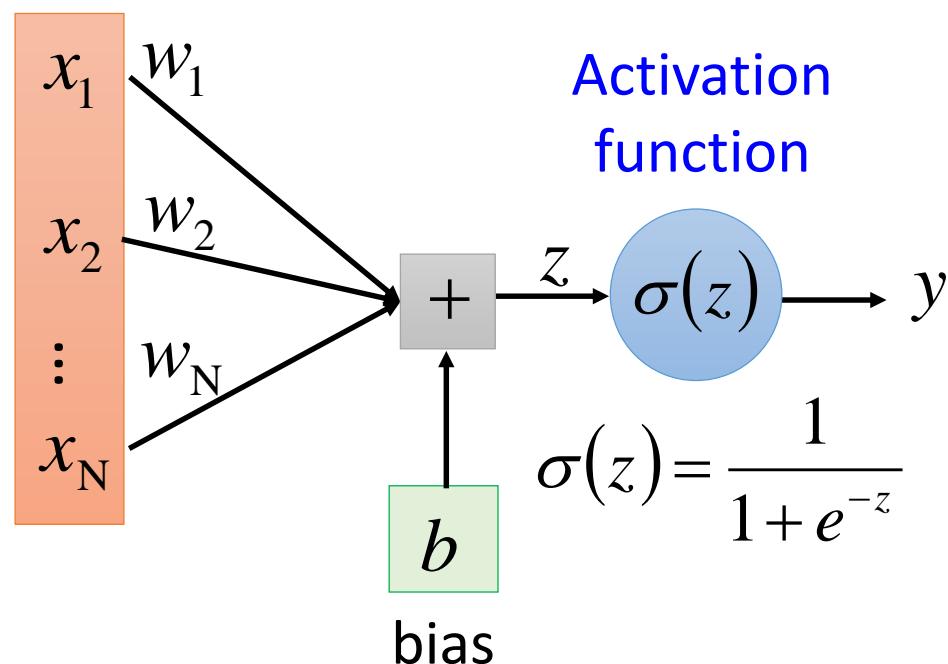
$$g = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \text{ (line 10)}$$

You cannot compute the gradients
of C because it is not a scalar

Single Neuron

Single Neuron

- First, let's implement a neuron



In this stage, let's assume the model parameters w and b are known

```
1 import theano
2 import theano.tensor as T
3 import random
4
5 x = T.vector()
6 w = T.vector()
7 b = T.scalar()
8
9 z = T.dot(w, x) + b
10 y = 1/( 1 + T.exp( -z ) )
11
12 neuron = theano.function(
13     inputs=[x,w,b],
14     outputs=[y]
15 )
16
17 w = [-1, 1]
18 b = 0
19 for i in range(100):
20     x = [random.random(), random.random()]
21     print x
22     print neuron(x,w,b)
```

$$y = \text{neuron}(x; w, b)$$

Single Neuron – Shared Variables

- In the last example, a neuron is a function with input x , w and b .
- However, we usually only consider x as *input*. w and b are *model parameters*.
 - It would be more intuitive if we only have to write “neuron(x)” when using a neuron
 - The model parameters w and b still influence `neuron(.)`, but in an implicit way.
- In Theano, the model parameters are usually stored as *shared variables*.

```
1 import theano
2 import theano.tensor as T
3 import random
4 import numpy
5
6 x = T.vector()
7 w = theano.shared(numpy.array([1., 1.]))
8 b = theano.shared(0.)
9
10 z = T.dot(w, x) + b
11 y = 1 / (1 + T.exp(-z))
12
13 neuron = theano.function(
14     inputs=[x],
15     outputs=y
16 )
17
18 print w.get_value()
19 w.set_value([0., 0.])
20
21 for i in range(100):
22     x = [random.random(), random.random()]
23     print x
24     print neuron(x)
```

w and b are declared as shared variables.

Initial value
(The shared variables are not symbols.)

x is the only input
The function can access the shared variables.

To get or change the values of the shared variables, you have to use "get_value()" and "set_value()".

Single Neuron – Training

- Define a cost function C
- Then compute $\left[\frac{\partial C}{\partial w_1}, \frac{\partial C}{\partial w_2}, \dots, \frac{\partial C}{\partial w_N} \right]$ and $\frac{\partial C}{\partial b}$

```
5 x = T.vector()
6 w = theano.shared(numpy.array([-1., 1.]))
7 b = theano.shared(0.)
8
9 z = T.dot(w, x) + b
10 y = 1 / ( 1 + T.exp( -z ) )
11
12 neuron = theano.function(
13     inputs=[x],
14     outputs=y
15 )
16
17 y_hat = T.scalar()
18 cost = T.sum((y-y_hat)**2)
19
20 dw, db = T.grad(cost, [w, b])
21
22 gradient = theano.function(
23     inputs=[x, y_hat],
24     outputs=[dw, db]
25 )
```

Reference output value

Define Cost

Computing Gradients

Declare the function for
computing gradients

Single Neuron – Gradient Descent

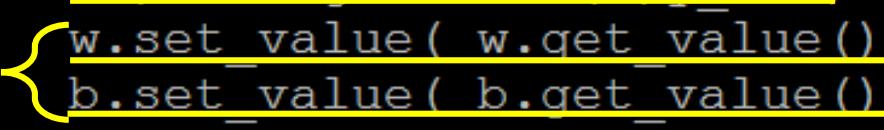
$$w_1 \leftarrow w_1 - \eta \frac{\partial C}{\partial w_1}, \dots, w_N \leftarrow w_N - \eta \frac{\partial C}{\partial w_N}, b \leftarrow b - \eta \frac{\partial C}{\partial b}$$

Tedious Way:

```
27 x = [1, -1]
28 y_hat = 1
29 for i in range(100):
30     print neuron(x)
31     dw, db = gradient(x, y_hat)
32     w.set_value( w.get_value() - 0.1 * dw )
33     b.set_value( b.get_value() - 0.1 * db )
34     print w.get_value(), b.get_value()
```

Update Parameters

Compute gradient



Line 31: use the function `gradient` (defined in the last page) to compute the gradients

Line 32, 33: use the gradients to update the model parameters

Single Neuron – Gradient Descent

$$w_1 \leftarrow w_1 - \eta \frac{\partial C}{\partial w_1}, \dots, w_N \leftarrow w_N - \eta \frac{\partial C}{\partial w_N}, b \leftarrow b - \eta \frac{\partial C}{\partial b}$$

Effective Way:

```
22 gradient = theano.function(
23     inputs=[x,y_hat],
24     updates=[(w,w-0.1*dw), (b,b-0.1*db)]
25 )
26
27 x = [1, -1]
28 y_hat = 1
29 for i in range(100):
30     print neuron(x)
31     gradient(x,y_hat)
32     print w.get_value(),b.get_value()
```

Line 24: updates=“a list of pairs”

Each pair is in the form of (shared-variable, an expression).

Whenever this function runs, it will replace the value of each shared variable with the result of the expression.

Single Neuron – Gradient Descent

$$w_1 \leftarrow w_1 - \eta \frac{\partial C}{\partial w_1}, \dots, w_N \leftarrow w_N - \eta \frac{\partial C}{\partial w_N}, b \leftarrow b - \eta \frac{\partial C}{\partial b}$$

Effective Way:

```
23 def MyUpdate(paramters, gradients) :  
24     mu = 0.1  
25     parameters_updates = \  
26     [ (p , p - mu * g) for p,g in izip(paramters,gradients) ]  
27     return parameters_updates  
28  
29 gradient = theano.function(  
30     inputs=[x,y_hat],  
31     updates=MyUpdate([w,b],[dw,db])  
32 )
```

In deep learning, usually sophisticated update strategy is needed.

In this case, you may want to use a function to return the pair list for parameter update.

What is izip?

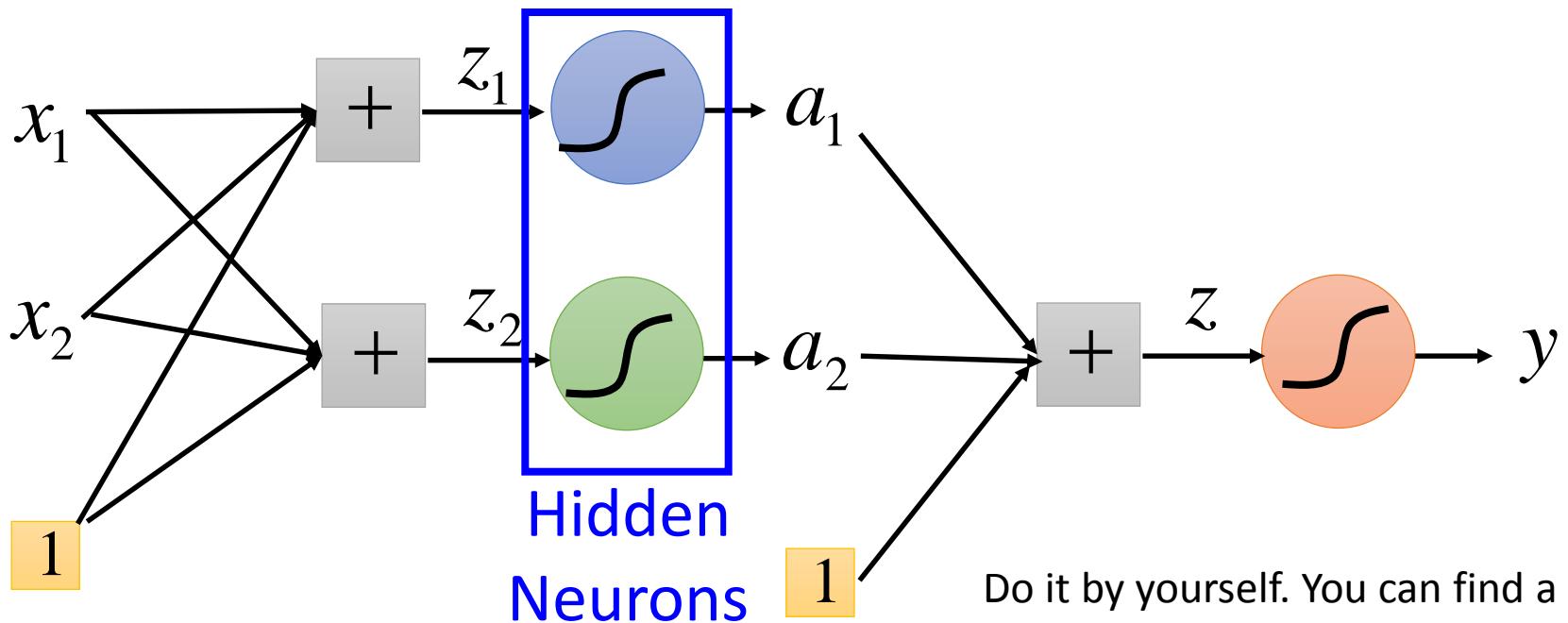
<https://docs.python.org/2/library/itertools.html#itertools.izip>

Tiny Neural Network

XOR gate

Can you use three neurons to simulate an XOR gate?

Input		Output
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



XOR gate

```
1 import theano
2 import theano.tensor as T
3 import numpy
4 from itertools import izip
5
6 x = T.vector()
7 w1 = theano.shared(numpy.random.randn(2))
8 b1 = theano.shared(numpy.random.randn(1))
9 w2 = theano.shared(numpy.random.randn(2))
10 b2 = theano.shared(numpy.random.randn(1))
11 w = theano.shared(numpy.random.randn(2))
12 b = theano.shared(numpy.random.randn(1))
13
14 a1 = 1/( 1 + T.exp( -1 * (T.dot(w1,x) + b1) ) )
15 a2 = 1/( 1 + T.exp( -1 * (T.dot(w2,x) + b2) ) )
16 y = 1/( 1 + T.exp( -1 * (T.dot(w,[a1,a2]) + b) ) )
17
18 y_hat = T.scalar()
19 cost = - (y_hat*T.log(y)+(1-y_hat)*T.log(1-y)).sum()
20
21 dw,db,dw1,db1,dw2,db2 = T.grad(cost,[w,b,w1,b1,w2,b2])
```

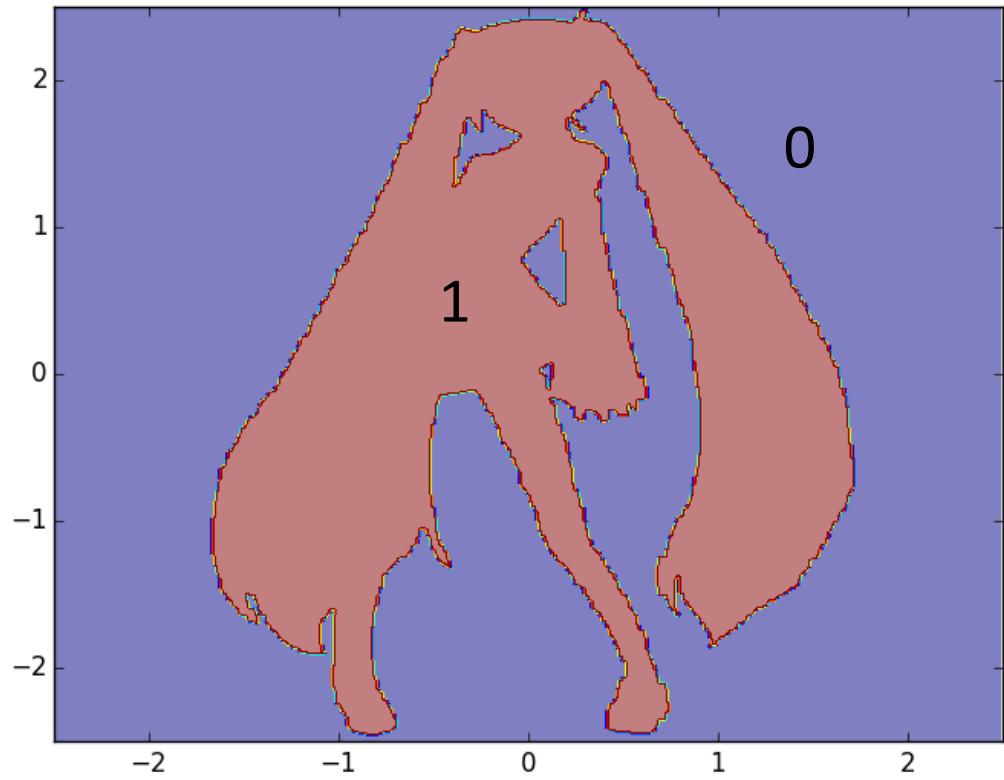
XOR gate

```
23 def MyUpdate(paramters,gradients) :
24         mu = 0.01
25         parameters_updates = \
26             [ (p,p-mu*g) for p,g in izip(paramters,gradients) ]
27         return parameters_updates
28
29 g = theano.function(
30 inputs=[x,y_hat],
31 outputs=[y,cost],
32 updates=MyUpdate([w,b,w1,b1,w2,b2],[dw,db,dw1,db1,dw2,db2])
33 )
34
35 for i in range(100000):
36     y1,c1 = g([0, 0], 0)
37     y2,c2 = g([0, 1], 1)
38     y3,c3 = g([1, 0], 1)
39     y4,c4 = g([1, 1], 0)
40     print c1+c2+c3+c4
41     print y1,y2,y3,y4
42
```

(continued from previous page)

Neural Network

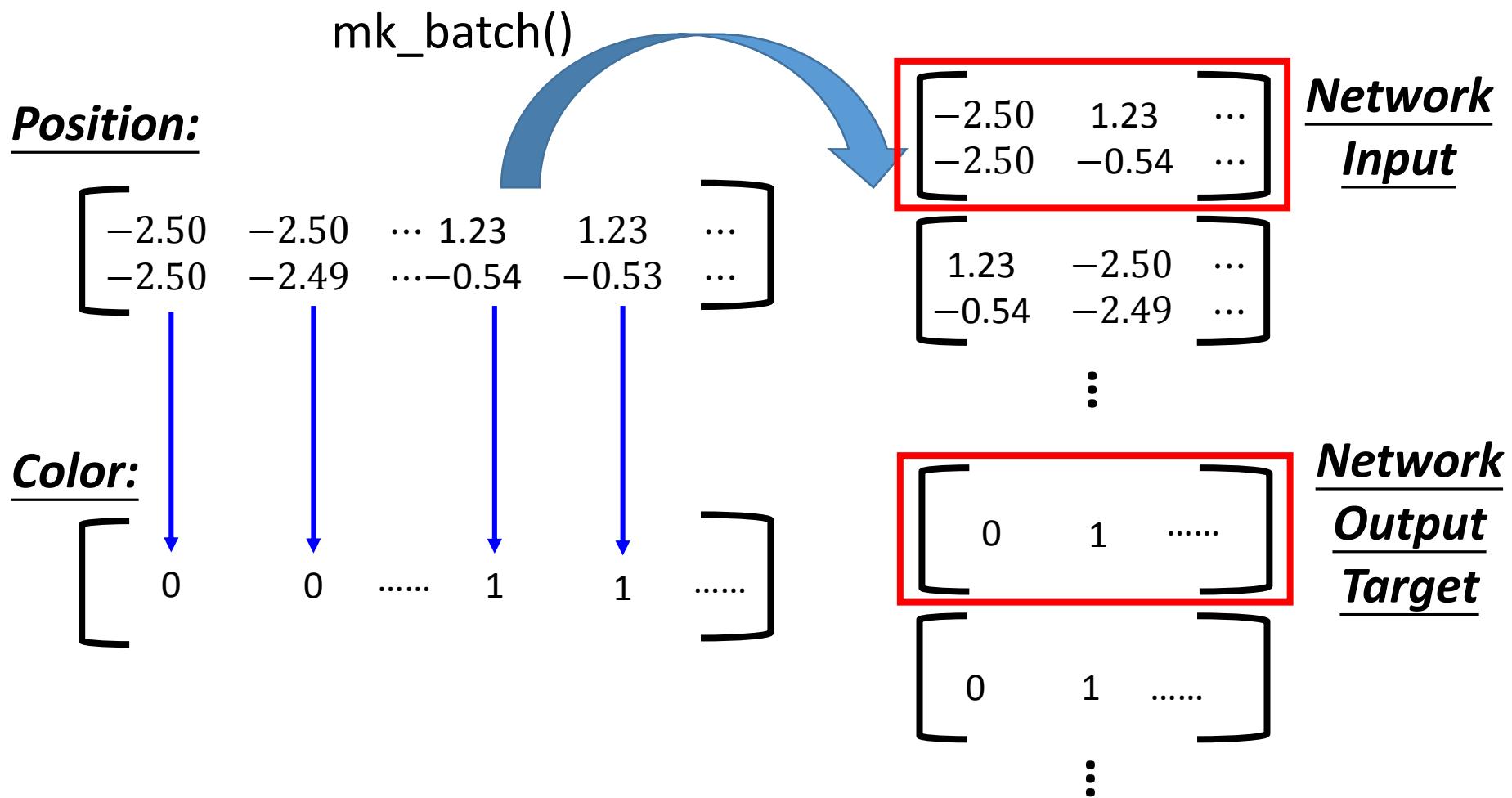
Function of Miku



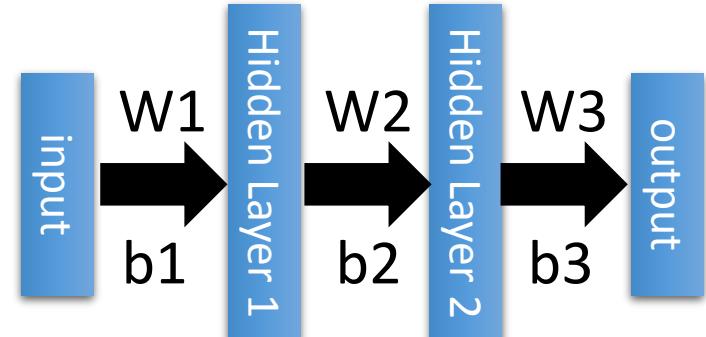
http://speech.ee.ntu.edu.tw/~tlkagk/courses/MLDS_2015_2/theano/miku

(1st column: x, 2nd column: y, 3rd column: output (1 or 0))

Make Minibatch



Defining Network



- Declare network Input
 - `x = T.matrix('input',dtype='float32')`
- Declare network output
 - `y_hat = T.matrix('reference',dtype='float32')`
- Declare network parameters
 - `W1 = ... (matrix), W2 = ..., W3 = ...`
 - `b1 = ... (vector), b2 = ... (vector), b3 = ... (vector)`
 - `parameters = [W1,W2,W3,b1,b2,b3]`

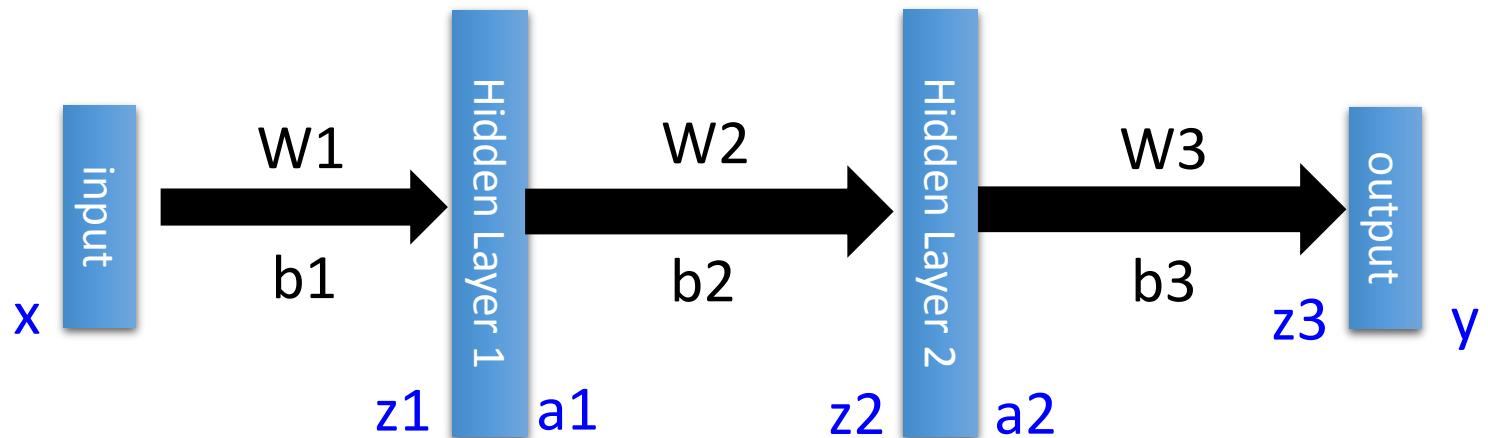
$$\begin{bmatrix} -2.50 & 1.23 & \dots \\ -2.50 & -0.54 & \dots \end{bmatrix}$$

Network
Input

$$\begin{bmatrix} 0 & 1 & \dots \end{bmatrix}$$

Network
Output
Target

Defining Network



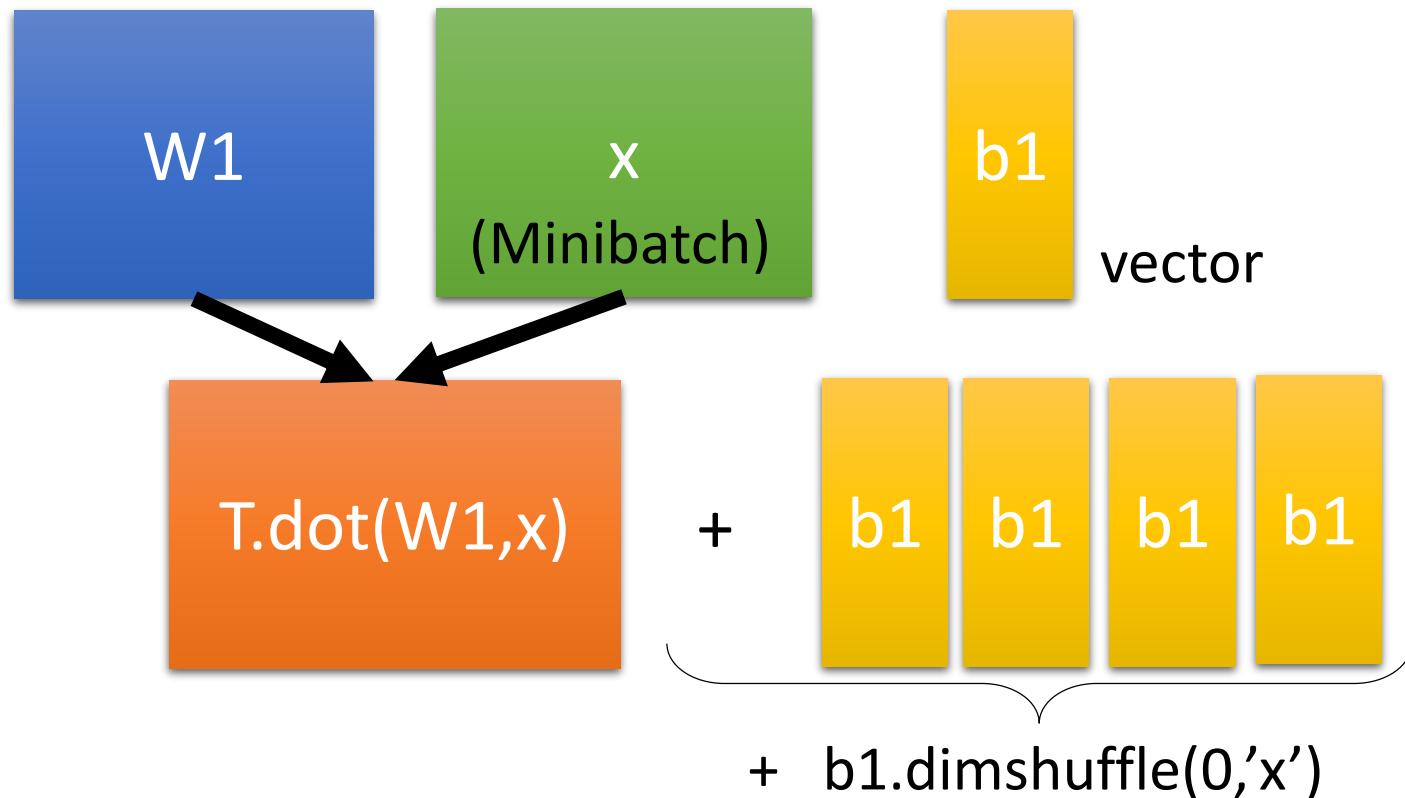
```
z1 = T.dot(W1, x) + b1.dimshuffle(0, 'x')
a1 = 1/(1+T.exp(-z1))
z2 = T.dot(W2, a1) + b2.dimshuffle(0, 'x')
a2 = 1/(1+T.exp(-z2))
z3 = T.dot(W3, a2) + b3.dimshuffle(0, 'x')
y = 1/(1+T.exp(-z3))
```

Minibatch

Ref:

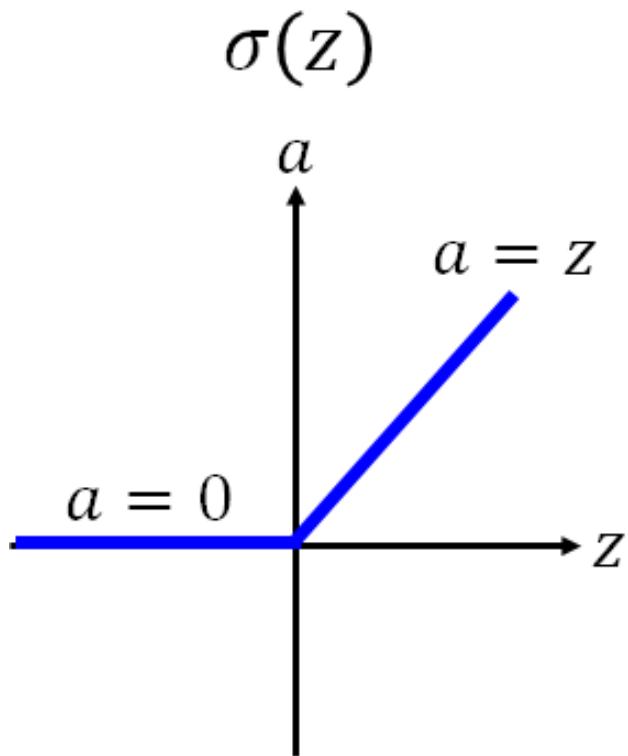
<https://theano.readthedocs.org/en/rel-0.6rc3/library/tensor/basic.html>

```
z1 = T.dot(W1, x) + b1.dimshuffle(0, 'x')
```



More about computing: http://videolectures.net/deeplearning2015_coates_deep_learning/

Activation Function - ReLU



If $z < 0$: $a = 0$
else: $a = z$



$a = \text{T.switch}(z < 0, 0, z)$

Other Approaches:

$a = \text{T.maximum}(z, 0)$

$a = z * (z > 0)$

Declaring Functions

```
cost = T.sum( (y - y_hat)**2 ) / batch_size
```

```
gradients = T.grad(cost, parameters)
```

Where is backpropagation?

Theano will do backpropagation automatically

```
train = theano.function(
    inputs=[x, y_hat],
    updates=MyUpdate(parameters, gradients),
    outputs=cost
)
```

```
test = theano.function(
    inputs=[x],
    outputs=y
)
```

Training & Testing

- Training:

100 epochs

Generating Batches

```
for t in range(100):
    cost = 0
    X_batch, Y_hat_batch \
        = mk_batch(X_all, Y_hat_all, data_size, batch_number)
    for i in range(batch_number):
        cost += train(X_batch[i], Y_hat_batch[i])
    cost /= batch_number
    print cost
```

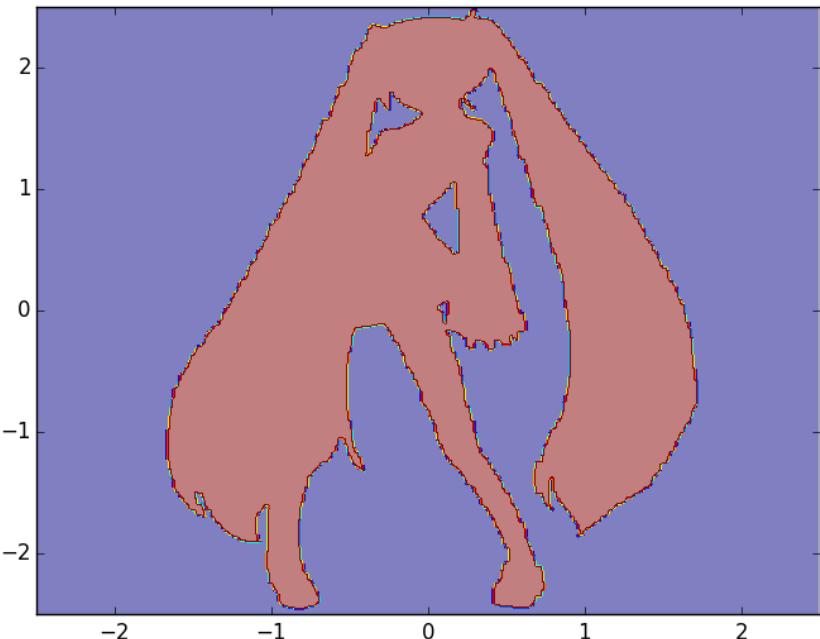
In real learning problem, you should use a validation set to control when to stop.

Update parameters after seeing each batch

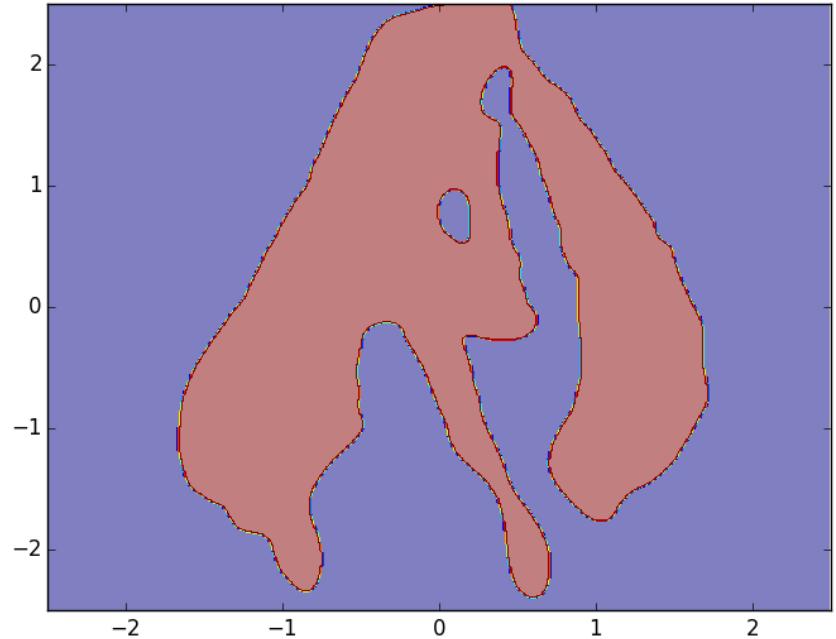
- Testing: $Y = \text{test}(X_{\text{all}})$

Results

Reference



Network Output



Network Structure: 2-1000-1000-1
Batch size: 1000
Activation function: sigmoid
Learning rate: 0.01 (fixed)

Configuration

Configuration

- There are three ways to set the configuration of Theano:
 - 1. modify `$HOME/.theanorc`
 - 2. set `theano.config.<property>` in your python code
 - 3. use `THEANO_FLAGS` when running your python code
 - E.g. `THEANO_FLAGS=mode=FAST_RUN,device=gpu`
`python YourCode.py`
- Ref:
<http://deeplearning.net/software/theano/library/config.html>

Configuration - GPU

- `test_gpu.py`

Multiplying two
10000 X 10000
matrices

```
1 import theano
2 import theano.tensor as T
3 import numpy
4 import time
5
6 X = T.matrix()
7 Y = T.matrix()
8 Z = T.dot(X, Y)
9 f = theano.function([X, Y], Z)
10
11 x = numpy.random.randn(10000, 10000)
12 y = numpy.random.randn(10000, 10000)
13 tStart = time.time()
14 z = f(x, y)
15 tEnd = time.time()
16 print "It cost %f sec" % (tEnd - tStart)
```

Command: `python test_gpu.py`

Output: `It cost 21.249996 sec`

If the machine has multiple GPUs,
using gpu0, gpu1 to select a card.

Command: `THEANO_FLAGS=device=gpu python test_gpu.py`

Output: `It cost 21.045571 sec` GPU is not helpful? Why?

Configuration - GPU

```
1 import theano
2 import theano.tensor as T
3 import numpy
4 import time
5
6 X = T.matrix(dtype='float32')
7 Y = T.matrix(dtype='float32')
8 Z = T.dot(X, Y)
9 f = theano.function([X, Y], Z)
10
11 x = numpy.random.randn(10000, 10000).astype(dtype='float32')
12 y = numpy.random.randn(10000, 10000).astype(dtype='float32')
13 tStart = time.time()
14 z = f(x, y)
15 tEnd = time.time()
16 print "It cost %f sec" % (tEnd - tStart)
17
```

Command: `THEANO_FLAGS=device=gpu python test_gpu.py`

Output: `It cost 0.843893 sec` More than 20 times faster

Why?

Ref: http://deeplearning.net/software/theano/tutorial/using_gpu.html

More about Configuration

- mode=DEBUG_MODE
 - Theano provides information to help you debug
- profile=true
 - Theano will analyze your program, showing a breakdown of how much is spent on each operation.

To Learn More ...

- Theano Exercises
 - https://github.com/goodfeli/theano_exercises
- Theano Tutorial
 - <http://deeplearning.net/software/theano/tutorial/index.html#tutorial>

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